Team Round

Lexington High School

December 8, 2018

Potpourri [200]

1. Evaluate $1+3+5+\cdots+2019$.
2. Evaluate $1^2-2^2+3^2-4^2+\cdots+99^2-100^2$.
3. Find the sum of all solutions to $|2018+|x-2018||=2018$.
4. The angles in a triangle form a geometric series with common ratio $\frac{1}{2}$. Find the smallest angle in the triangle.
5. Compute the number of ordered pairs (a,b,c,d) of positive integers $1 \le a,b,c,d \le 6$ such that $ab+cd$ is a multiple of seven.
6. How many ways are there to arrange three birch trees, four maple, and five oak trees in a row if trees of the same species are considered indistinguishable.
7. How many ways are there for Mr. Paul to climb a flight of 9 stairs, taking steps of either two or three at a time?
8. Find the largest natural number x for which x^x divides 17!
9. How many positive integers less than or equal to 2018 have an odd number of factors?
10. Square MAIL and equilateral triangle LIT share side IL and point T is on the interior of the square. What is the measure of angle LMT?
11. The product of all divisors of 2018^3 can be written in the form $2^a \cdot 2018^b$ for positive integers a and b. Find $a+b$.
12. Find the sum all four digit palindromes. (A number is said to be palindromic if its digits read the same forwards and backwards.
13. How ways are there for an ant to travel from point $(0,0)$ to $(5,5)$ in the coordinate plane if it may only move one unit in the positive x or y directions each step, and may not pass through the point $(1,1)$ or $(4,4)$?
14. A certain square has area 6. A triangle is constructed such that each vertex is a point on the perimeter of the square. What is the maximum possible area of the triangle?
15. Find the value of ab if positive integers a,b satisfy $9a^2-12ab+2b^2+36b=162$.
16. $\triangle ABC$ is an equilateral triangle with side length 3. Point D lies on the segment BC such that $BD=1$ and E lies on AC such that $AE=AD$. Compute the area of $\triangle ADE$.
17. Let $A_1, A_2, \cdots, A_{10}$ be 10 points evenly spaced out on a line, in that order. Points B_1 and B_2 lie on opposite sides of the perpendicular bisector of A_1A_{10} and are equidistant to l. Lines $B_1A_1, \cdots, B_1A_{10}$ and $B_2A_1, \cdots, B_2A_{10}$ are drawn. How many triangles of any size are present?
18. Let $T_n = 1+2+3\cdots+n$ be the nth triangular number. Determine the value of the infinite sum

$$\sum_{k\ge 1} \frac{T_k}{2^k}.$$

19. An infinitely large bag of coins is such that for every $0.5 < p \le 1$, there is exactly one coin in the bag with probability p of landing on heads and probability $1-p$ of landing on tails. There are no other coins besides these in the bag. A coin is pulled out of the bag at random and when flipped lands on heads. Find the probability that the coin lands on heads when flipped again.

20. The sequence $\{x_n\}_{n\ge 1}$ satisfies $x_1 = 1$ and

$$(4 + x_1 + x_2 + \cdots + x_n)(x_1 + x_2 + \cdots + x_{n+1}) = 1$$

for all $n \ge 1$. Compute $\left\lfloor \frac{x_{2018}}{x_{2019}} \right\rfloor$.